The use of Computational Fluid Dynamics for an investigation of the effects of acceleration on the aerodynamics of a missile

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Abstract

In this study, we discuss how the block-structured CFD-code EURANUS handles arbitrarily shifting reference frames, and we provide the results of our validation efforts. We also provide the calculated outcomes of two applications employing accelerated missiles with generic setups. We demonstrate that acceleration has a considerable impact on the resistance of waves. It is also shown that the vortices created by strakes turn dramatically. The importance of accounting for acceleration effects in the computations is shown by these findings.

Keywords: acceleration, aerodynamics, drag, CFD, vortex

Introduction

In the past, CFD has mostly been used for stationary objects with time-dependent models of transient phenomena, and for systems revolving at a constant angular velocity, including air flows, turbines, and spinning blades. Aeroelastic models, moving control surfaces, rotor blades, and product release are only few examples of systems that use relative motion. It is necessary to address arbitrarily shifting reference frames if one is to accurately forecast the behavior of accelerating and maneuvering flying bodies.

The aerodynamics of missiles and the computation of dynamic derivatives are two examples of the kind of applications that inspired this study. It is worth noting that 4th gen missiles can complete a turn with an angular acceleration on the order of 10 g to 100 g (where g is the acceleration due to gravity). Projectiles that experience thrusts in the range of 100 g to 500 g are also attracting attention. An acceleration of this magnitude has the potential to significantly alter the flow field.

It is generally understood that there is a well-developed body of theory for dynamic derivatives subject to geometry and speed restrictions (see, for example, Nielsen [1]). Weinacht and Sturek [29] are credited with performing the first aerodynamic characterization of a spinning kinetic energy projectile. Using the use of quasi-stationary ALE (Arbitrary-Lagrangian-Eulerian) models [12], Cormier et al. [3] determined the damping derivatives of roll, yaw, and pitch. For example, in a simulation using multi-level Cartesian grids, Murman et al. [19] found that ALE approaches were beneficial for calculating the aerodynamic stresses and roll-averaged pitch induced by canard dither. Dynamic derivatives have been calculated using automated differentiation by Park et al. [23], [22], while Green et al. [10] used low-order panel techniques with automatic differentiation to get F-16XL aircraft dynamic derivatives. Forcing oscillations may also be modeled using an alternate approach, such as Murman's [18] reduced-frequency technique. Several authors (e.g. Shaw and Qin [26]) have tried to represent the oscillating blades seen in forward helicopter flight using computational fluid dynamics (CFD), and this is often done by introducing an elastic deformation of the grid.

Overset or adaptive grids (such as those cited by Cenko et al. [2]) have also been used to simulate the aerodynamics of objects speeding on arbitrarily predefined or 6DOF trajectories, a technique often used to represent store separation in the transonic domain. These approaches are only practical for near-field interactions since a backdrop grid hundreds of meters in length is needed to cover the trajectory corridor for transonic missiles performing rotations at 50 g to 100 g. As for linear acceleration, Roohani and Skews [25] have done some work. The alternative approach of integrating over the large datasets that have been amassed in aerodynamic parameter research (e.g. Murman et al. [20]) has been used to generate extended trajectories, but this can only be done if the population of parameter space is dense enough.

In contrast to the aforementioned approaches, the goal of this study is to provide direct load prediction during free-form maneuvers that may entail substantial linear acceleration or thrust, as well as variable rotational acceleration (as at the start or finish of a turn). Some of the features sought for are forced-oscillation models and the ability to calculate dynamic derivatives.

Theory

A mathematical tool box for the analytic treatment of non-stationary problems with relatively moving frames, including an extended vector analysis in an arbitrary number of dimensions, has been formulated by Löfgren [17], and forms the foundation for the following finite volume formulation. Consider a fixed, inertial reference frame Σ, referred to as the absolute frame (Figure 1). Consider also a moving reference frame Σ′, which may be accelerating in an arbitrary fashion, and which is referred to as the relative frame.

Notation is introduced to distinguish between vectors in Σ viewed in Σ', and *vice versa*. A general vector a, with components in Σ viewed in Σ, is denoted by a when viewed from Σ'. For example, if a is constant in time but Σ' rotates, a must have rotating components. A vector a in Σ' is expressed as a when viewed from Σ. The rotation of Σ' relative to Σ is denoted by the orthogonal transform U.

(1)

Let r be the displacement vector of the origin of Σ′ interpreted in Σ. Coordinates x in Σ′ are related to coordinates x in Σ by the transformation

$$x = r + U \cdot x \tag{2}$$

and its inverse. For time derivatives we are able [7] to define a rotation vector ω by

$$\omega \times (U \cdot a) = \frac{\partial U}{\partial t} \cdot a \tag{3}$$

Differentiating with respect to time we obtain absolute and relative velocities respectively:

$$v = \dot{r} + U \cdot (v + \omega \times x) = \dot{r} + v + \omega \times x$$

$$v = -r + v - \omega \times x \tag{4}$$

The relative velocity field u between the two frames is defined by $u = v - v$ which leads to

$$u = \dot{r} + \omega \times x \tag{5}$$

Spatial derivatives and frame transformations can then be derived [7]. Note that density and pressure are invariant under transformation. The conservation equations may be expressed in Σ or Σ′. Conventionally, CFD calculations are carried out in the relative frame Σ′. The coordinate system and grid attached to the object in Σ′ are shown in fig. 1. To illustrate the formulation, in the absolute frame

we have

$$\frac{\partial}{\partial t}\int_V \rho v dV + \oint_{\partial V} (\rho v \otimes (v - \underline{u}) + pI) \cdot d\underline{S} = 0 \qquad (6)$$

where the tensor product with components $[a \otimes b]_{ij} = a_i b_j$ has been used and viscous fluxes have been neglected for the purposes of illustration. In the relative frame, it is possible to show that the momentum conservation equation can be written [7]

$$\frac{\partial}{\partial t}\int \rho \underline{v} dV + \oint (\rho \underline{v} \otimes \underline{v} + pI) \cdot d\underline{S} = -\int \rho \left(\frac{\partial \dot{r}}{\partial t} + \frac{\partial \underline{\omega}}{\partial t} \times \underline{x} + 2\omega \times \underline{v} + \omega \times (\omega \times \underline{x}) \right) dV$$

(7)

v ∂v v

The complicated source terms in the non-inertial frame can be interpreted as the fictitious forces of Batchelor [1] and Greenspan [11]. From the left, the terms represent the fictitious forces of translational acceleration (if *U* captures all rotation), angular acceleration, Coriolis effects, and centrifugal effects in the non-inertial frame. In the absolute frame Σ, no source terms appear.

In a numerical implementation, there is a choice between using the absolute frame formulation and the absolute velocities *v* or the relative frame formulation and velocities $\underline{v}$. The relative velocities $\underline{v}$ are small close to solid bodies for viscous calculations where the grid tends to be very fine. Hence, they reduce the risk of inaccuracies arising from truncation error. The presence of significant source terms, however, may change the properties of the numerical scheme and the conservative character is lost.

The absolute formulation and the use of absolute velocities *v* can be integrated very easily into schemes which already use stretchable coordinates or moving grids. Modifications are required to boundary conditions, and since absolute velocities *v* are frequently difficult for practitioners to interpret, the transformation of flow fields to the relative frame is a necessary tool. An estimate indicates that truncation errors in the near field would be acceptably reduced by the use of double precision. The absolute formulation was therefore chosen for the present work. A precise description of the computational procedure is given in Forsberg [7], and related background material can be foundin previous papers [8], [9].

Implementation

For the numerical implementation we have used EURANUS, which is a general Euler and Navier-Stokes solver for structured, and possibly non-matching, multi-block meshes. A cell centred finite volume approach is used with central differences, symmetric Total Variation Diminishing (TVD) or upwind TVD flux difference splitting. An explicit Runge-Kutta local time-stepping is used for steady state calculations, and an implicit time-integration with dual time-stepping is used for the time accurate computations. To enhance the convergence implicit residual smoothing (IRS) and full approximation storage (FAS) multigrid are used.

EURANUS already has grid-stretching introduced for aero-elastic purposes which greatly simplify the introduction of moving reference frames. This code is extensively verified ([24], [5], [15], [6], [28]). The existing routines supporting the implementation of stretchable meshes for aero-elastic calculations made the implementation of absolute instead of relative velocities straightforward. No extra terms which might ruin the conservative form of the equations have been added [5]. The modifications to the boundary condition routines and the rest of the program required by the presence of absolute velocities were also found to be minor.

The input variables are $\dot{\underline{r}}$, the velocity of the Σ′ origin seen in Σ′, and ω , the rotation of the

moving frame about its origin, seen in Σ′. Input is provided by the user at arbitrary successive times in the input file. Cubic interpolation then provides values at intermediate times corresponding to the solver time steps. Although the implementation is fully viscous only Euler cases are shown in this paper.

Validation case 1: rotating plate

In order to validate the implementation of the rotation transform, the existing extensively verified version of EURANUS intended for coordinate systems rotating with constant angular velocity has been used for comparison. This version includes rotation effects as source terms on the right-hand side of the Navier-Stokes equations. From this point in the paper, variables are written without the framenotation in Σ′.

As a simple test case incorporating the steady revolution of a simple flat plate of zero thickness is constructed and rotated about a centre at distance *R* (see Figure 2). The circular trajectory is followed by a pivot point either halfway along the plate (centre case) or at the leading edge of the plate (skew case). The revolution angular velocity is 40 s^{-1}, the revolution radius 5 m and the speed of the plate 200 ms^{-1}. A comparison between the pressures on the plate for the two cases is shown when convergence has been reached in Figure 3. No significant differences between the results are seen.

I. Validation case 2: constant velocity airfoil

The simplest validation for linear velocity terms to be performed is a test of Galilean invariance with no frame acceleration present, in which the pressure profile across an airfoil travelling at constant velocity ($-u_x$, $-u_y$, *0*), modelled in the absolute frame with stagnant far-field boundary conditions, is compared with the pressure profile across an airfoil modelled in the relative frame with free stream boundary conditions (u_x, u_y, *0*). No acceleration is present in these cases.

The conditions chosen are Mach 0.8 with angle of attack α = 1.25º. A two-dimensional grid with 257 x 65 points is used. The chord length L is 1.0 m; characteristic far field boundaries are placed at 25 chord lengths away from the slip airfoil surface in the dimensions modelled. A second order central difference scheme was used with Jameson dissipation [14], [13]. An implicit five stage Runge-Kutta scheme with backward Euler time differencing, 5 W-cycle multi-grid levels and residual smoothing were used. Pressure coefficients C_p were compared for relative and absolute frames and are very similar for the two cases.

II. Validation case 3: oscillating airfoil

We consider the oscillating airfoil case published by Landon [16]. The experimental pressure and force measurements have been extensively used to evaluate time-dependant solvers [30], [4]. The entire grid oscillates rigidly and the grid moves at Mach 0.755. At far-field boundaries, conditions of stagnant flow (zero velocity) are imposed. The angle of attack of the airfoil varies as a function of time t as

$$\alpha(t) = \alpha_0 + \alpha_1 \sin \omega t \quad (8)$$

with α_0 = 0.016º and α_1 = 2.51º. The angular velocity ω is chosen such that the dimensionless frequency k = 0.0814, with the definition $k = \omega L / 2u_\infty$ where L is the scale length, in this case the chord length, and u_∞ is the relative speed of the airfoil and the flow. The airfoil is oscillated about its quarter-chord point. A two-dimensional O-mesh with 129 points on the airfoil surface and 33 points to the outer boundary was used [27].

The chord length is 1.0 m (in contrast to that of Landon [16], where the chord length in the experiment was 0.1016 m). Characteristic far field boundaries are placed at 25 chord lengths away from the airfoil, and the external flow is specified as stagnant. In the absolute frame, the airfoil is moved at speed $u = -u_\infty$ and pitched so that the angle of attack is given by equation (6). A steady field for α_0 = 0.016º was provided as initialisation.

The normal force coefficients C_N and pitching moment C_m are shown as a function of α inFigure 4, and pressure coefficients C_p for selected α are shown in Figure 5. The agreement between the absolute frame model and experimental results is reasonable and consistent with other calculations [10].

III. Application case: rapidly accelerating missile

As a first application, we consider a simple missile configuration with a flare subject to very rapid (4500 ms^{-2}) linear acceleration along the major axis (Figure 6). At these extreme accelerations no experiments are yet available. The main practical interest is in the impact of the acceleration on the drag coefficient C_d.

We compare with steady state calculations performed at a given set of velocities.

The results of Euler calculations for the simplified missile are shown in Figure 7. The drag coefficient has been normalized with respect to the instantaneous dynamic pressure. The main observed impact of the acceleration on the drag is that the transonic maximum is reduced by approximately 20% and moved above the speed of sound, having a maximum value at approximately Mach 1.2.

IV. Application case: vortex behaviour in turn

A motivating interest application is the increasing manoeuvrability of missiles. As illustrated in Figure 8, vortices from nose, canards, body or strakes which interact with fins may move as a result of significant acceleration in a turn, and at certain angles of attack may change either fin disruption or pressure footprints on the body. The vortices in this case are generated along sharp-edged zero- thickness strakes rather than by separation along the curved surfaces of the hemisphere-cylinder body,

Figure 9.

For a typical speed of 600 ms^{-1}, a pitch rate of q=1 s^{-1} corresponds to a turn radius of 60 m, and a transverse acceleration of 600 ms^{-2} or approximately 60 *g*, where *g* is the acceleration due to gravity.

For a 2 m typical length *L*, the ratio of *L* to turn radius *R* is about 1/30, indicating that centrifugal effects would be small but significant. The Rossby number U/2L*q* is 150.

The total length is 2.000 m, the diameter is 0.100 m, and the strakes project 0.010 m from the surface. The origin is at the nose of the hemisphere and the coordinate *x* extends along the main axis. We define the angle of attack *α* as the kinematic angle between the direction of flow and the *x* axis in the body axis system, and the pitch angle *θ* as the attitudinal angle between a fixed axis in an inertial system, which may be referred to as x_1, and the *x* axis in the body axis system. The pitch rate or angular velocity, *q*, is defined as $\dot{\theta}$. The object executes a circle with α constant and constant *q*. From derivatives with respect to *q* of the pitching moment C_m and the lift coefficient C_N we obtain the dynamic derivatives C_{mq} and

C_{Nq} respectively.

Normal force coefficients and pitching moment coefficients for varying q are shown in Figure 10 for α = 15º. Lines have been fitted to the points excluding $q < -10$ s^{-1} and the dynamic derivatives $C_{mq} = \partial C_m/\partial q$ and $C_{Nq} = \partial C_N/\partial q$ are the slopes.

We compare the flow field at α = 15º and no rotation, q = 0 s^{-1}, with the flow field for a sample rotation, q = -5 s^{-1}. The upper strake vortices can easily be traced from their inception and their propagation backwards until they eventually merge with the wake. To quantify the pressure change, we consider the difference field $\Delta p = p|_{q=-5} - p|_{q=0}$.

In the wake, a clear displacement exists in the top strake vortex under rotation, and the displacement is upwards, towards the centre of revolution; the contrast appears in Figure 11 (a) and (b) at x = 2.5 m. The difference field Δp shows changes of the order of 10^4 Pa. The pressure changes on the windward side of the body account for the dominant contribution to lift and pitching moment with negative q in Figure 10.

V. Conclusions

We develop and implement a prediction method applicable to the aerodynamics of arbitrary manoeuvres, formulated in the absolute frame. The absolute frame formulation has the advantage that no source terms need be incorporated, and conservation is automatic. The method was easy to implement in the existing well-validated Navier-Stokes code EURANUS, since partly-moving meshes have already been incorporated for aeroelastic purposes. We have validated the absolute frame method using pressure coefficients over a constant velocity transonic NACA0012 test case, a rotating plate and the oscillating airfoil data of Landon [16].

A flare has been accelerated through Mach 1 and the drag coefficient results compared with those obtained at constant velocity. The main observations that the transonic maximum wave drag is reduced by approximately 20% and that the maximum value occurs at approximately Mach 1.2. To investigate the effect of turn rate, we considered vortices being generated along sharp-edged strakes on a hemisphere-cylinder. In the wake, a clear displacement exists in the top strake vortex under rotation towards the centre of revolution. These effects are of interest because vortices from nose, canards, body or strakes which interact with fins will move as a result of significant acceleration in a turn, andat certain angles of attack may change either fin disruption or pressure footprints on the body. Also, the dynamic derivatives C_{mq} and C_{Nq} are easily available from the method.

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